Department of Physics, IIT, Delhi

PHL110: Fields and waves

Major Examination

All Questions are compulsory

You must define all symbols (other than stated in the questions) and state known results which you use.

Date: 04-05-2009

Full Marks: 40 **Time: 2 Hours**

Useful Physical constants:

$e = 1.602 \times 10^{-19}$ Coulomb, $1\,eV = 1.602 \times 10^{-19}$ J, $m_e = 9.109 \times 10^{-31}$ Kg,

$\hbar = 6.58 \times 10^{-16}$ eV-s, Or, $\hbar = 1.054 \times 10^{-34}$ J-s

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}\, N/A^2$, c=$3.0 \times 10^8$ m/s

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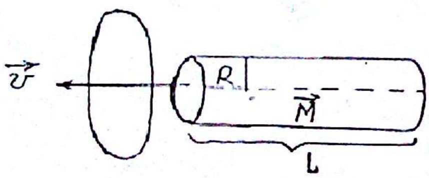
1. (a) Charge q is uniformly distributed over a conducting spherical shell of radius R_1. A point charge q_0 is placed at the center of the shell. Find the work done by the electrostatic forces in the expansion of the shell to a radius $R_2 > R_1$ (4)

(b) A long cylindrical magnet of length L and radius R carries a uniform magnetization $\vec{M}$ parallel to its axis. It passes at constant velocity v through a circular wire ring of slightly larger diameter, as shown in the figure. Draw qualitative graphs of the flux through the ring and the emf induced in the ring, as a function of time, and explain them. (4)



2. A point charge q is embedded at the center of a sphere of linear dielectric material with susceptibility χ_e and radius R. Calculate the electric field vector, the polarization vector and the bound charge density at a point inside the sphere. What is the total bound charge at the surface? (8)

3. (a) A long narrow slit of width b is covered by a thin glass slide of thickness t and refractive index n, such that the glass slide covers half the slit (form 0 to b/2).

PTO

Show that the Fraunhofer diffraction pattern of the slit, when illuminated by a plane wave of wavelength λ, incident normally from one side of the slit, is given by

$$I(\theta) = I_0 \frac{\sin^2\beta}{\beta^2} \cos^2(\beta + \phi/2)$$

where θ is the angle that the diffracted wave makes with the normal to the slit, $\beta = \frac{\pi b}{2\lambda}\sin\theta$ and ϕ is the phase difference induced by the glass slide. (4)

(b) Using the above result, draw qualitatively the Fraunhofer diffraction patterns for $t = 0.6\mu m$ and $t = 1.2\mu m$, with $b = 50\mu m$, $n = 1.5$ and the wavelength of light equal to 600 nm. Mark the positions of the minima of the diffraction pattern in each case. (4)

4. Consider one dimensional motion of a particle in the following potential:

$$V(x) = \begin{cases} 0 & 0 < x < a, \\ \infty & \text{elsewhere.} \end{cases}$$

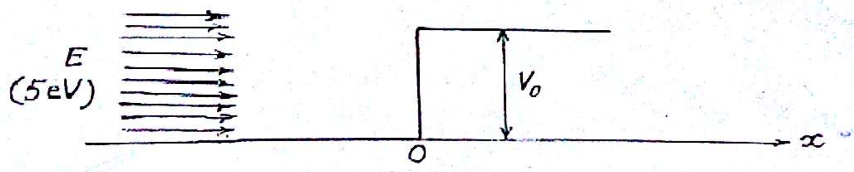
The normalized eigenfunctions and the corresponding eigenvalues (eigen-energies) for the stationary states are, respectively:

$$\phi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right), \quad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad n = 1, 2, 3, \ldots$$

(a) Using these, show that $\langle xp\rangle - \langle px\rangle = i\hbar$ for the n-th stationary state, where $\langle\cdot\rangle$ stands for the expectation value of the quantity inside the brackets. (5)

(b) Use the uncertainty relation between the position and the corresponding momentum to compute the velocities of an electron, moving in a box of width $a = 10^{-10}m$, and that of a metallic sphere of mass 1 g, moving in a box of width 10 cm. (you can use the values $\hbar c = 200$ MeV fm and $m_e c^2 = 0.5$ MeV, where MeV stands for megaelectronvolts and fm stands for femtometer (10^{-15}m). (3)

5. Consider a beam of electrons, with energy $5eV$, moving from $-\infty$ to the right. This beam is incident on a potential barrier of height $V_0 = 7eV$ (see the figure below).



Obtain expressions for the reflected and the transmitted amplitudes in terms of the incident amplitude. Calculate the reflection coefficient (R) and the transmission coefficient (T) and show that $R + T = 1$. (4+4)